Math 525

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1 Continuous RV

$$\text{Prob}(a \leq X \leq b) = \int_a^b f_X(x)dx$$

- $f_X \geq 0$
- $\int_{-\infty}^{\infty} f_X(x)dx = 1$
- sum formulas

$F_X(x)$ cumulative distribution.

$$F_X(x) = \int_{-\infty}^{x} f_X(t)dt.$$

Example 1.1. Uniform$[a, b]$

$$\chi_{unif[a,b]}(x) = \begin{cases} 0, & x \notin [a,b] \\ 1, & x \in [a,b] \end{cases}, \ f_{unif[a,b]} = \frac{1}{a-b}\int_{-\infty}^{x} \chi(t)dt$$

Example 1.2.

$$f_X(x) = \begin{cases} 0, & \\ \lambda e^{-\lambda x}, & \lambda > 0 \end{cases}$$

Exponential RV, analogues to geometric distributions

$$\mathbb{E}(X) = \frac{1}{\lambda}.$$

Example 1.3. T-distribution

$$\Gamma(s) = \int_0^{\infty} x^s e^{-d} dx$$

Generalities on $f_X(x)$.

Independence of 2 RV's, $\mathbf{X}$, $\mathbf{Y}$. For continuous distributions, $\mathbf{X}$, $\mathbf{Y}$ are independent iff

$$f_{\mathbf{X},\mathbf{Y}}(x,y) = f_{\mathbf{X}}(x) f_{\mathbf{Y}}(y).$$

$f_{\mathbf{X}}(x)$ is the marginal:

$$f_{\mathbf{X}}(x) = \int_{-\infty}^{\infty} f_{\mathbf{X},\mathbf{Y}}(x,y) dy.$$

So

$$\begin{aligned} \mathbb{P}(a \leq \mathbf{X} \leq b, c \leq \mathbf{Y} d) &= \int_a^b \int_c^d f_{\mathbf{X},\mathbf{Y}}(x,y) dy dx \\ &= \left(\int_a^b f_{\mathbf{X},\mathbf{Y}}(x,y) dx \right) \left(\int_c^d f_{\mathbf{X},\mathbf{Y}}(x,y) dy \right) \\ &= \mathbb{P}(a \leq \mathbf{X} \leq b) \mathbb{P}(c \leq \mathbf{Y} d). \end{aligned}$$

Converse: