

Gluing the Edges of a Polygon

Gluing the opposite edges of a square together yields a torus. Similarly (though this requires serious distortion of the polygon) one can glue together pairs of edges of a polygon P with any even number of sides, to obtain a compact surface. To begin such a gluing process, one must decide how to pair the edges up, and there are many ways to do this. The genus of the surface that is obtained depends on this pairing.

Example: Say that we label the edges of a square, in clockwise order, as e_1, \dots, e_4 . To obtain a torus, which has genus 1, one glues e_1 to e_3 and e_2 to e_4 . One could also glue e_1 to e_2 and e_3 to e_4 , and the result would be topologically a sphere, which has genus 0.

Assignments:

1. Explain the conditions under which the surface obtained by gluing will be orientable.
2. Set up a computer program that will determine all ways to glue the edges of a polygon so as to obtain a compact orientable surface, and that will compute the genus of that surface.
3. Use the computer program to investigate the surfaces that can be obtained. Sample questions you might study: What are the possible genera? What is the maximal genus? What is the probability that, if the pairing is chosen at random, the genus will be maximal?